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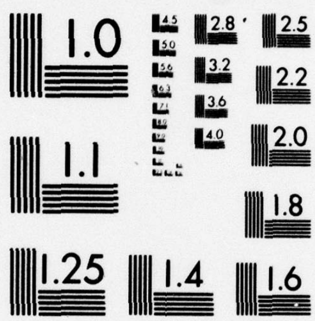
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6 A THEORY FOR PORTIONS OF THE ENERGY SPECTRUM AND FOR INTERMITTENCY OF FINE-SCALE TURBULENCE.

By

10 Robert R. Long

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By

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A THEORY FOR PORTIONS OF THE ENERGY SPECTRUM AND FOR INTERMITTENCY OF FINE-SCALE TURBULENCE

ABSTRACT

The paper contains a theory for two ranges of the energy spectrum, $k_e \ll k \ll k_m$ and $k_m \ll k \ll k_s$, where k is wave number k_e is the wave number of the energy-containing eddies, $k_m = \lambda^{-1}$ where λ is Taylor's microscale and $k_s = \eta^{-1}$ where η is the Kolmogorov length. The results are obtained by recognizing the existence of a "mesoregion" in wave-number space in which the wave number is of order λ^{-1} and assuming a new "inner" behavior of the spectrum function for larger k based on the two scales η and λ. Reynolds number similarity is assumed as a first approximation for smaller k (outer region) and an assumption that the mesoregion and the outer region overlap leads to infinite series for the spectrum function in each of the two ranges. The forms reduce to the $k^{-\frac{5}{3}}$-law in both ranges in the limit as k/k_e and k_s/k get large. Universal constants may be chosen to yield excellent agreement with the data for a tidal channel.

The paper concludes with some conjectures on the intermittency of fine-scale turbulence and the geometry of the fine structure. It is suggested that the intermittency factor is proportional to $R_\lambda^{-\frac{1}{2}}$.

A THEORY FOR PORTIONS OF THE ENERGY SPECTRUM AND FOR INTERMITTENCY OF FINE-SCALE TURBULENCE

1. Introduction.

We may briefly summarize the existing theories of the energy spectrum function E for the higher wave numbers. As discussed by Tennekes and Lumley (1971, p. 265), the classical theory of Kolmogorov and Obukhov may be obtained by assuming that E has an "outer" behavior for smaller values of k varying with k, the integral length scale of the turbulence, k_e^{-1}, and the dissipation function, ϵ, and an "inner" behavior for larger k varying with ϵ, ν and k. A matching procedure similar to that used by Isakson (1937) and Millikan (1938) for shear flow then leads to the classical form

$$E = C_1 \epsilon^{\frac{2}{3}} k^{-\frac{5}{3}}, \quad (k_e \ll k \ll k_s) \tag{1}$$

where C_1 is a universal constant, $k_s = \eta^{-1}$ and η is the Kolmogorov length $\nu^{\frac{3}{4}}/\epsilon^{\frac{1}{4}}$. More recently, Kolmogorov (1962), Obukhov (1962) and Gurvich and Yaglom (1967) have proposed

$$E = C_2 \epsilon^{\frac{2}{3}} k^{-\frac{5}{3}} \left(\frac{k}{k_e}\right)^{-\frac{\mu}{9}} \tag{2}$$

where μ is a new constant. It is generally believed that the constant $\mu/9$ is small and that this explains why the data points lie close to a line of $-\frac{5}{3}$ slope on log-log paper. Experimental evidence based on measurements of flatness factors suggest values of μ in the range 0.40–0.85 (Gibson and Masiello, 1972, Ueda and Hinze, 1975). There is debate about the validity of (1) and (2) (Kraichnan, 1974), but Mandelbrot (1971) believes that the assumption behind (2) that the local values of ϵ are log-normally distributed is untenable.

In a recent paper, the author (Long, 1979) advanced a new theory of turbulent shear flow which proposes that there is a second boundary layer (mesolayer) near the wall of a thickness proportional to Taylor's microscale, λ, for the core region. This profoundly affects the matching procedure of Izakson and Millikan and leads to new predictions for various quantities. More importantly for present purposes, "bursts" seem to originate in the mesolayer and this suggests that a similar region should exist in wave-number space for large k of "thickness" $k_s \sim \lambda^{-1}$ in which the viscous dissipation region is imbedded and in which there is intermittency, and that the matching to produce a prediction for the inertial subrange should take this into account. We show in Section 2 that there is such a mesoregion and the new matching is presented in Section 3. As in shear flow, the process leads to a predicted behavior of E in two ranges: $k_s \ll k \ll k_s$, and $k_e \ll k \ll k_s$, which we call, respectively, the viscous and inertial subranges.

2. Identification of the Mesoregion.

We may show that there is a very special region in wave-number space between the inertial and viscous ranges in which neither the inner quantity ν nor the outer quantity k_e may be neglected, precisely as in the mesolayer in shear flow (Long, 1979). We show this for the special case of decaying, isotropic, self-preserving turbulence, but it seems likely on physical grounds that the existence of the mesoregion is quite general. We need the following (Korneyev and Sedov, 1976):

$$\lambda = \alpha_1 \nu^{\frac{1}{2}} t^{\frac{1}{2}}, \quad \sigma = at^{-n}, \quad \epsilon = \alpha_2 \sigma^3 k_e \tag{3}$$

$$\lambda = k_e^{-1} R_L^{-\frac{1}{2}}, \quad \frac{d\epsilon}{dt} = -\frac{(2n+1)\epsilon}{t} \tag{4}$$

where $R_L = \sigma/\nu k_e$ is the Reynolds number, σ is the rms velocity and α_1, α_2, a, n are independent of time. We also need the energy equation (Hinze, 1975, p. 215)

$$\frac{\partial E}{\partial t} = T - 2\nu k^2 E \tag{5}$$

where T is the energy transfer from smaller to larger wave numbers and $D = 2\nu k^2 E$ is the dissipation. In the inertial range, $k \sim k_e$,

$$E \sim \epsilon^{\frac{2}{3}} k_e^{-\frac{5}{3}}, \quad \frac{\partial E}{\partial t} \sim \frac{E}{t}, \quad D \sim \nu k_e^2 E \tag{6}$$

In the viscous range, $k \sim k_s$,

$$E \sim \frac{\epsilon}{\nu k_s^3}, \quad \frac{\partial E}{\partial t} \sim \frac{E}{t}, \quad D \sim \nu k_s^2 E \tag{7}$$

Thus $(\partial E/\partial t)/D$ is of order R_L in the inertial range and of order $R_L^{-\frac{1}{2}}$ in the viscous range. The situation is as portrayed in Fig. 1. At smaller wave-numbers, the time rate-of-change (decrease) of energy is close to the non-linear transfer (loss) of energy and the dissipation is small. Scaled on outer variables, these quantities are of order 1, 1, R_L^{-1}, respectively. At large wave numbers the time rate-of-change (decrease) of energy is very small and the non-linear transfer (gain) of energy is very closely balanced

by viscous dissipation. These quantities are of order $R_L^{-\frac{5}{4}}$, $R_L^{-\frac{3}{4}}$, $R_L^{-\frac{3}{4}}$, respectively. Obviously, since T is zero at an intermediate wave number, $\partial E/\partial t$ and D are of the same order in a range of wave numbers of order

$$k_s \sim (\nu t)^{-\frac{1}{2}} \sim \lambda^{-1} \sim k_e R_L^{\frac{1}{2}} \tag{8}$$

so that an effort to find a region between the viscous and inertial regions where <u>both</u> ν and k_e can be neglected, as in the classical arguments, seems doomed to failure because the two regions of classical theory are separated by a region in which <u>neither</u> ν nor k_e can be neglected.

3. Derivation of the Spectral Behavior.

As suggested by the theory of shear flow (Long, 1979) and the arguments of Section 2, we express the spectrum for large k as

$$E = \frac{\epsilon}{\nu k^3}\left[F_+(\xi) + h_1(R_L)F_{*1}(\hat{\xi}) + h_2(R_L)F_{*2}(\hat{\xi}) + \dots\right] \tag{9}$$

where $\xi = k_s/k = \epsilon^{\frac{1}{4}}/\nu^{\frac{3}{4}}k$, $\hat{\xi} = \xi/R_L^{\frac{1}{4}}$. We assume that only the F_+-term survives at fixed ξ as $R_L \to \infty$. We see that $\hat{\xi} \sim 1$ corresponds to the meso-region. Useful relations are

$$\hat{\xi} = \xi R_L^{-\frac{1}{4}}, \quad \xi = \zeta R_L^{\frac{3}{4}} \alpha_2^{\frac{1}{4}}, \quad \hat{\xi} = \zeta R_L^{\frac{1}{2}} \alpha_2^{\frac{1}{4}} \tag{10}$$

where $\zeta = k_e/k$ and we have used (3) and (4). In the outer region, we express E as a function of outer variables ϵ, k_e and k to the first approximation, so that

$$E = \epsilon^{\frac{2}{3}} k^{-\frac{5}{3}}[f_0(\zeta) + \ell_1(R_L)f_1(\zeta) + \ell_2(R_L)f_2(\zeta) + \dots] \tag{11}$$

We will use Eq. (5) and, therefore, we need the energy-transfer function T. We write

$$T = (\epsilon\nu)^{\frac{3}{4}}[H_+(\xi) + m_1(R_L)H_{*1}(\hat{\xi}) + m_2(R_L)H_{*2}(\hat{\xi}) + \ldots] \tag{12}$$

as the inner expansion and

$$T = \epsilon k^{-1}[g_0(\zeta) + p_1(R_L)g_1(\zeta) + p_2(R_L)g_2(\zeta) + \ldots] \tag{13}$$

as the outer expansion. Eq. (5) yields, after some algebra, the following three equations

$$H_+ - 2\xi F_+ = 0 \tag{14}$$

$$\begin{aligned} m_1 H_{*1} + m_2 H_{*2} + \ldots - 2\hat{\xi} R_L^{\frac{1}{4}}[h_1 F_{*1} + h_2 F_{*2} + \ldots] = \\ -\alpha\hat{\xi}^3 R_L^{\frac{1}{4}}[(2n+1)(F_+ + \tfrac{1}{4}F_+'\xi + h_1 F_{*1} + h_2 F_{*2} + \ldots)] - \alpha\hat{\xi}^3 R_L^{\frac{1}{4}}\left[\frac{h_1}{2}F_{*1}'\hat{\xi} + \frac{h_2}{2}F_{*2}'\hat{\xi} + \ldots\right] \\ -\alpha\hat{\xi}^3 R_L^{\frac{1}{4}}[(2n-1)(R_L h_1' F_{*1} + R_L h_2' F_{*2} + \ldots)] \end{aligned} \tag{15}$$

where $\alpha = \alpha_1^2/\alpha_2^{\frac{1}{2}}$. The matching yields

$$\alpha_2^{-\frac{1}{3}}\zeta^{-\frac{4}{3}}(f_0 + \ell_1 f_1 + \ell_2 f_2 + \ldots) = R_L(F_+ + h_1 F_{*1} + h_2 F_{*2} + \ldots) \tag{16}$$

$$\alpha_2^{\frac{1}{4}}\zeta(g_0 + p_1 g_1 + p_2 g_2 + \ldots) = R_L^{-\frac{3}{4}}(H_+ + m_1 H_{*1} + m_2 H_{*2} + \ldots) \tag{17}$$

A solution of (14)-(17) is

$$\ell_1 = R_L^{-1},\ p_1 = R_L^{-1},\ h_1 = R_L^{-\frac{1}{3}},\ m_1 = R_L^{\frac{1}{4}-\frac{1}{3}} \tag{18}$$

$$F_+ = A_0\xi^{-\frac{4}{3}} + A_1\xi^{-\frac{8}{3}} + A_2\xi^{-\frac{12}{3}} + \ldots \tag{19}$$

$$f_0 = B_{00} + B_{01}\zeta^{\frac{2}{3}} + B_{02}\zeta^{\frac{4}{3}} + B_{03}\zeta^{\frac{6}{3}} + \ldots \tag{20}$$

$$f_1 = B_{10}\zeta^{-\frac{6}{3}} + B_{11}\zeta^{-\frac{4}{3}} + B_{12}\zeta^{-\frac{2}{3}} + B_{13} + \ldots \tag{21}$$

$$f_2 = B_{20}\zeta^{-\frac{12}{3}} + B_{21}\zeta^{-\frac{10}{3}} + B_{22}\zeta^{-\frac{8}{3}} + B_{23}\zeta^{-\frac{6}{3}} + \dots \tag{22}$$

etc.

$$F_{*1} = C_{10}\hat{\xi}^{-\frac{4}{3}} + C_{11}\hat{\xi}^{-\frac{10}{3}} + C_{12}\hat{\xi}^{-\frac{16}{3}} + \dots \tag{23}$$

$$F_{*2} = C_{20}\hat{\xi}^{-\frac{2}{3}} + C_{21}\hat{\xi}^{-\frac{8}{3}} + C_{22}\hat{\xi}^{-\frac{14}{3}} + \dots \tag{24}$$

$$F_{*3} = C_{30} \quad + C_{31}\hat{\xi}^{-\frac{6}{3}} + C_{32}\hat{\xi}^{-\frac{12}{3}} + \dots \tag{25}$$

etc.

$$H_{*1} = E_{10}\hat{\xi}^{\frac{5}{3}} + E_{11}\hat{\xi}^{-\frac{1}{3}} + E_{12}\hat{\xi}^{-\frac{7}{3}} + \dots \tag{26}$$

$$H_{*2} = E_{20}\hat{\xi}^{\frac{7}{3}} + E_{21}\hat{\xi}^{\frac{1}{3}} + E_{22}\hat{\xi}^{-\frac{5}{3}} + \dots \tag{27}$$

$$H_{*3} = E_{30}\hat{\xi}^{\frac{9}{3}} + E_{31}\hat{\xi}^{\frac{3}{3}} + E_{32}\hat{\xi}^{-\frac{3}{3}} + \dots \tag{28}$$

etc.

$$g_0 = D_{00}\zeta^{\frac{2}{3}} + D_{01}\zeta^{\frac{4}{3}} + D_{02}\zeta^{\frac{6}{3}} + \dots \tag{29}$$

$$g_1 = D_{10}\zeta^{-\frac{4}{3}} + D_{11}\zeta^{-\frac{2}{3}} + D_{12} + \dots \tag{30}$$

$$g_2 = D_{20}\zeta^{-\frac{10}{3}} + D_{21}\zeta^{-\frac{8}{3}} + D_{22}\zeta^{-\frac{6}{3}} + \dots \tag{31}$$

etc.

Eqs. (14)-(17) provide relations among the various universal constants A_0, A_1, B_{00}, ... but no useful information for the final results of interest, which may be written

$$E = C_{0i}(\epsilon\nu^5)^{\frac{1}{4}}\left(\frac{k}{k_s}\right)^{-\frac{5}{3}}\left[1 + B_i\left(\frac{k}{k_s}\right)^{-\frac{2}{3}} + C_i\left(\frac{k}{k_s}\right)^{-\frac{4}{3}} + \dots\right] \tag{32}$$

for the inertial subrange, and

$$E = C_{0s}(\epsilon\nu^5)^{\frac{1}{4}}\left(\frac{k}{k_s}\right)^{-\frac{5}{3}}\left[1 - B_s\left(\frac{k_s}{k}\right)^{-\frac{4}{3}} + C_s\left(\frac{k_s}{k}\right)^{-\frac{8}{3}} - D_s\left(\frac{k_s}{k}\right)^{-\frac{12}{3}} + \dots\right] \tag{33}$$

for the viscous subrange. The coefficients in (32) and (33) were evaluated from the tidal channel data of Grant, Stewart and Moillet (1962). Tentative values

are $C_{01} = C_{02} = 0.5$, $B_1 = 5.0$, $B_2 = 4.575$, $C_2 = 6.05$, $D_2 = 1.0$. The contribution of the C_1-term in (32) is too small to measure. The comparison with the data is shown in Fig. 2. Notice that the $k^{-\frac{5}{3}}$ law represents the asymptotic behavior of the solutions in both ranges.

3. Intermittency of Fine-Structure Turbulence.

In Long (1979) the intermittency of the Reynolds stress, $-\overline{u'w'}$, near the wall in turbulent shear flow was found to be $\gamma \sim R_\lambda^{-\frac{1}{2}} \sim R_L^{-\frac{1}{4}}$ where the Reynolds numbers are based on quantities in the outer flow. If one assumes both

$$\overline{u'w'}\,\bar{u}_z \sim \nu\overline{(\nabla u')^2} \tag{34}$$

and

$$\overline{(u'w'\bar{u}_z)^2} \sim \nu^2\overline{(\nabla u')^4} \tag{35}$$

where z is distance from the wall, it follows that the intermittency of the fine structure of the turbulence near the wall follows the same law. This appears to be the same as the intermittency of the fine structure in the outer flow (Rao <u>et al.</u>, 1971) and we assume $\gamma \sim R_L^{-\frac{1}{4}}$ for the intermittency of the high wave-number turbulence of the present problem.

The geometry of the fine-structure regions has been investigated by Corrsin (1962), Tennekes (1968), Kuo and Corrsin (1971), Kuo and Corrsin (1972), and Badri Narayanan <u>et al.</u> (1977) in experiments in boundary layers and behind grids. Corrsin (1962) and others have suggested that the small eddies are in vortex sheets of thickness of order of Kolmogorov's microscale η and spacing k_e^{-1} or L. This yields an intermittency

$$\gamma \sim \eta/L \sim \frac{\nu^{\frac{3}{4}}}{\epsilon^{\frac{1}{4}}L} \sim R_L^{-\frac{3}{4}} \sim R_\lambda^{-\frac{3}{2}} \tag{36}$$

which is certainly too small. Tennekes suggested that the fine-structure was in the form of tubes of diameter η and spacing λ and this leads to

$$\gamma \sim \eta^2/\lambda^2 \sim R_L^{-\frac{1}{2}} \tag{37}$$

Badri Narayanan et al. attempted to measure the diameter and spacing and their data suggest a spacing of order λ and diameter ηR_L. Kuo and Corrsin's theories and measurements suggested that the tube-like structure was most likely and they found a diameter of order η.

We have seen that the term $\partial E/\partial t$ is of the order of the dissipation and the transfer terms for $k \sim \lambda^{-1}$. For larger k, $\partial E/\partial t$ is small and therefore represents the small difference between the vortex-stretching which efficiently produces small-scale eddies and viscous dissipation which efficiently destroys them. We may investigate the small difference by an artifice in which we imagine that we may remove all of the space between the fine-structure regions yielding a new space densely packed with these regions. The vortex stretching is now greatly reduced, and we have a balance (at least in order of magnitude) between viscous dissipation and $\partial E_s/\partial t$, i.e. the rate of decrease of the spectrum function E_s for these small eddies. E_s should be the same in the real and imagined motion because we have only eliminated the smaller wave numbers. The dissipation term will differ from that in Eq. (5) and we may obtain its form by noticing that in the Fourier representation in the new space we replace the quantity kx, for example, by $k(L_1/L_2)x$ where L_1 is the smallest length scale of the regions and L_2 is the distance between regions. Eq. (5) then may be written

$$\frac{\partial E_s}{\partial t} = -A\nu \frac{L_1^2}{L_2^2} k^2 E_s + T_1 \tag{38}$$

where A is a constant, and where T_1 is the transfer under the new conditions. It is reasonable that T_1 is no larger than the dissipation term and so is of the form $\nu^3 k^3 A_s H_s(k/k_s) L_1^2/L_2^2$ or smaller. Writing $E_s = \nu^2 k\varphi_s(k/k_s)$, we get

$$\frac{1}{2y}\frac{d\varphi_s}{dy} = A_1 R_L^{\frac{1}{2}} \frac{L_1^2}{L_2^2}[H_s(y) - \varphi_s(y)] \tag{39}$$

where $y = k/k_s$, and we again assume self-preservation. Eq. (39) requires

$$\frac{L_2}{L_1} \sim R_L^{\frac{1}{4}} \tag{40}$$

The contribution to the energy from the region of wave number space $k \sim k_s$ is

$$\gamma u_p^2 \sim \int_{k_s}^{\infty} \nu^2 k\varphi_s\left(\frac{k}{k_s}\right) dk \tag{41}$$

where u_p is the typical velocity in the fine-structure regions. This becomes

$$\gamma u_p^2 \sim \int_1^{\infty} \epsilon^{\frac{1}{2}} \nu^{\frac{1}{2}} y\varphi_s(y) dy \tag{42}$$

or

$$\gamma \sim R_L^{-\frac{1}{2}} \frac{\sigma^2}{u_p^2} \tag{43}$$

The final point in the argument is from the energy equation which yields

$$\frac{\sigma^2}{t} \sim \frac{\sigma^4}{K} \sim \nu\gamma \frac{u_p^2}{L_1^2} \tag{44}$$

Eqs. (40), (43) and (44) lead to

$$L_1 \sim \eta, \quad L_2 \sim \lambda \tag{45}$$

and the assumption that $\gamma \sim R_L^{-\frac{1}{4}}$ leads to

$$u_p^2/\sigma^2 \sim R_L^{-\frac{1}{4}} \tag{46}$$

The result that $\gamma = L_1/L_2$ suggests that the fine-structure regions are vortex sheets of thickness η and spacing λ or possibly ribbons of dimensions η and λ and spacing λ. Hinze (1975) and others consider ribbons the most likely possibility. The prediction that $\gamma \sim R_\lambda^{-\frac{1}{2}}$ suggests a flatness factor for the first derivative $F \sim R_\lambda^{\frac{1}{2}}$ and this is close to the experimental result $R_\lambda^{0.6}$ given by Kuo and Corrsin (1971).

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Mr. Ching-Sheng Chern analyzed the data and prepared Fig. 2.

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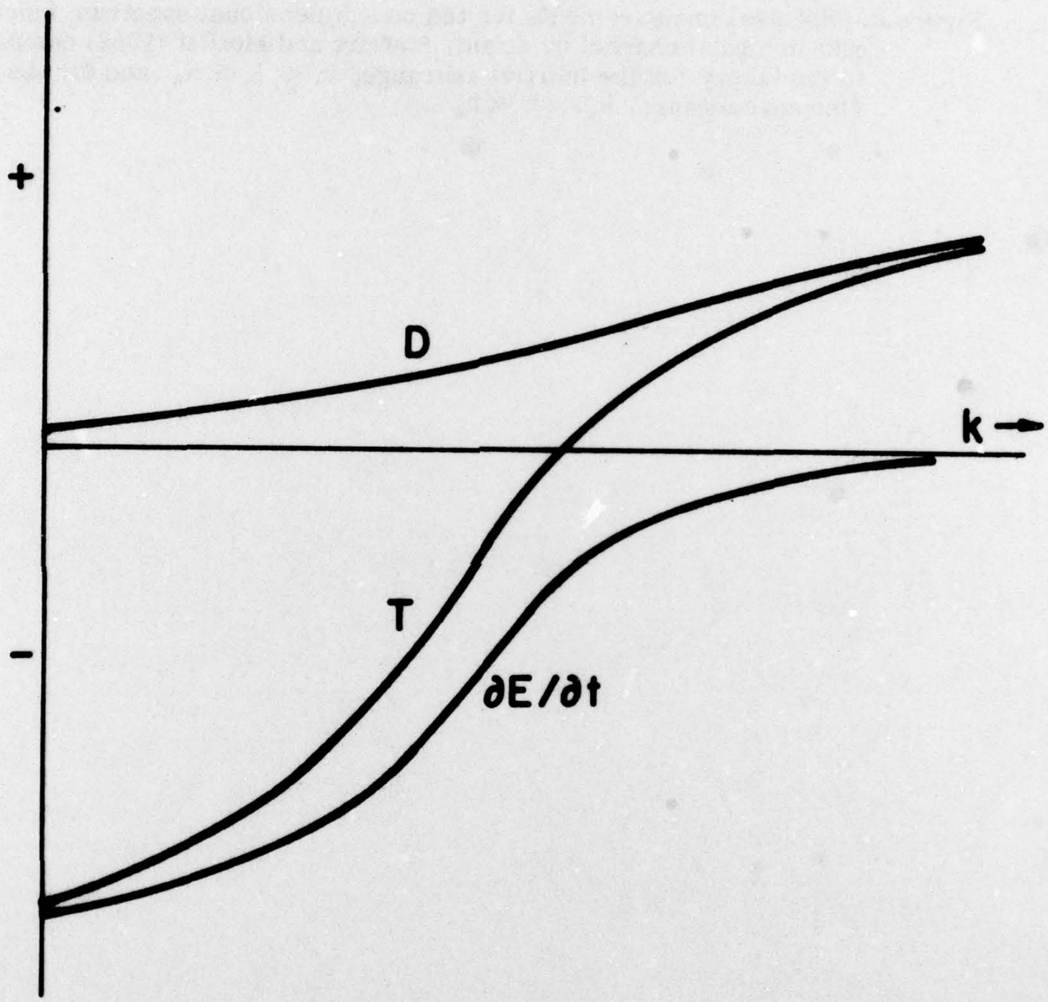
Figure 1. Schematic picture of energy balance in Eq. (5) as function of k.

Figure 2. Spectral measurements for the one-dimensional spectrum function $\varphi(k)$ in a tidal channel by Grant, Stewart and Moillet (1962) compared to the theory for the inertial subrange, $k_o \ll k \ll k_s$ and for the viscous subrange, $k_s \ll k \ll k_s$.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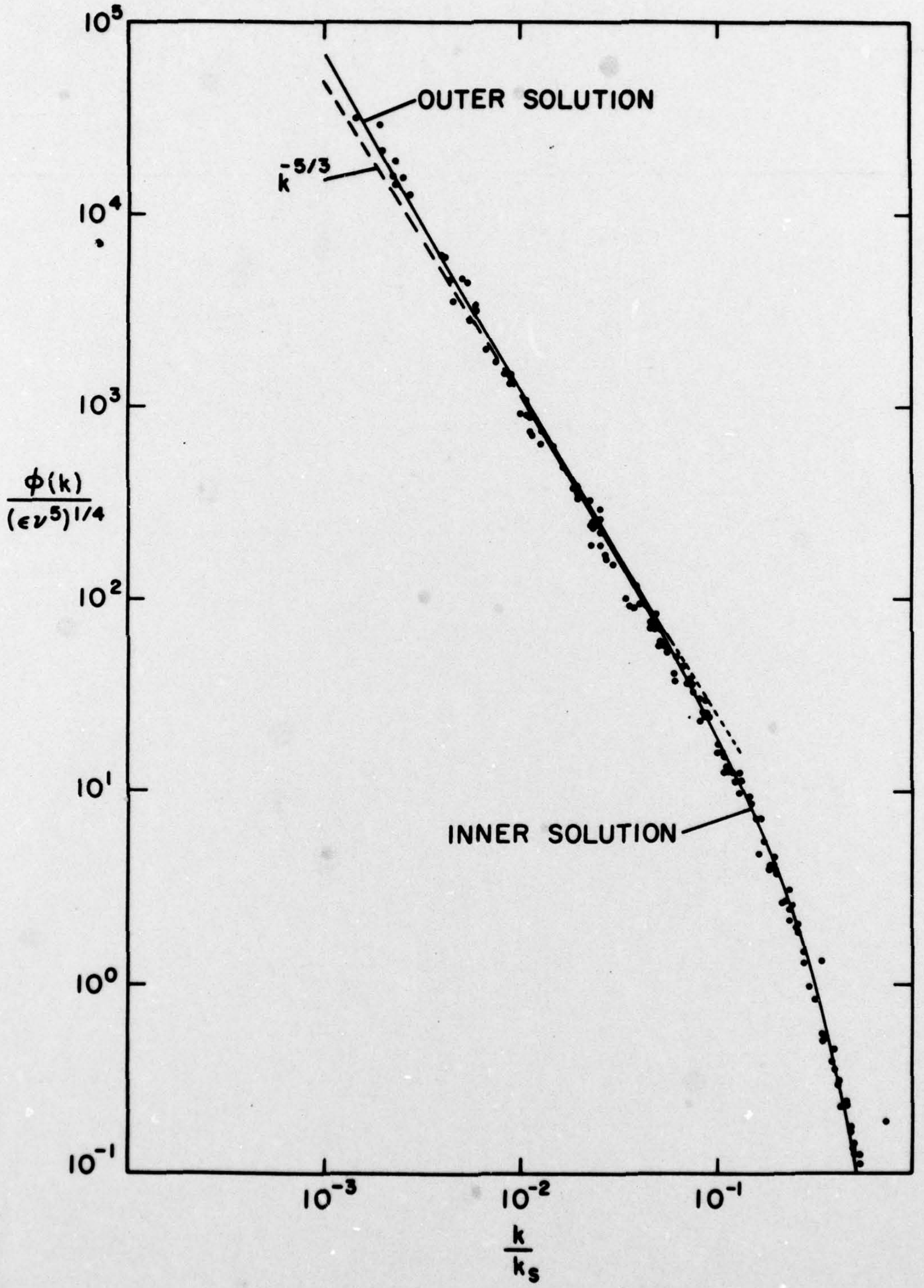
OUTER SOLUTION
$k^{-5/3}$
INNER SOLUTION
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10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-3}
10^{-2}
10^{-1}
$\frac{\phi(k)}{(\epsilon\nu^5)^{1/4}}$
$\frac{k}{k_s}$

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sub s   sub e   sub s   sub e   sub m

13. ABSTRACT

The paper contains a theory for two ranges of the energy spectrum, $k_e \ll k \ll k_m$ and $k_m \ll k \ll k_s$, where k is wave number, k_e is the wave number of the energy-containing eddies, $k_m = 1/\lambda$ where λ is Taylor's microscale and $k_s = 1/\eta$ where η is the Kolmogorov length. The results are obtained by recognizing the existence of a "mesoregion" in wave-number space in which the wave number is of order λ^{-1} and assuming a new "inner" behavior of the spectrum function for larger k based on the two scales η and λ. Reynolds number similarity is assumed as a first approximation for smaller k (outer region) and an assumption that the mesoregion and the outer region overlap leads to infinite series for the spectrum function in each of the two ranges. The forms reduce to the $k^{-5/3}$-law in both ranges in the limit as k/k_e and k_s/k get large. Universal constants may be chosen to yield excellent agreement with the data for a tidal channel.

lambda   eta   lambda   3 sub

sube   sub s   k to the -5/3 power

The paper concludes with some conjectures on the intermittency of fine-scale turbulence and the geometry of the fine structure. It is suggested that the intermittency factor is proportional to $R_\lambda^{-1/2}$.

square root of 1/R sub lambda

1/lambda

DD FORM 1 NOV 65 1473 (PAGE 1)

Security Classification

14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Turbulence						
Isotropic turbulence						
Spectrum						
Intermittency						
Taylor's microscale						
Kolmogorov length						

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